Biot-Barenblatt double porosity theory

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Objectives

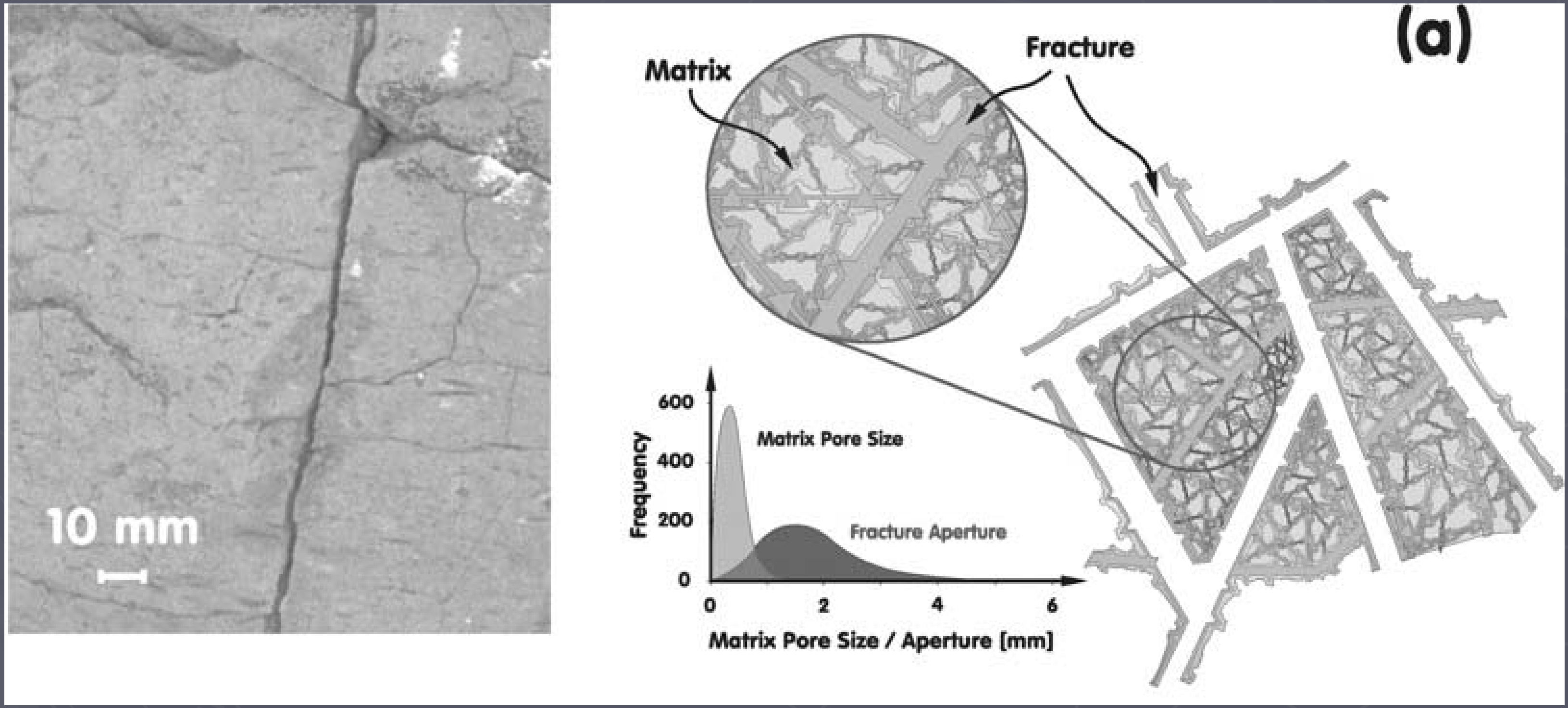
- Theoretical interpretation of the frequency dependent reflection from the hydrocarbon-bearing reservoir
- Asymptotic analysis of attenuation, reflection, transmission coefficients in order to obtain relatively simple expressions
  - "*All that is simple is false and all that is complex is useless*" // P.Valery
- Investigate the role of local formation permeability heterogeneities

Layout

- Poroelasticity + fluid flow = Biot's medium
- Fluid flow + local heterogeneities = Barenblatt's double porosity medium
- Poroelasticity + fluid flow + local heterogeneities = Biot-Barenblatt double porosity model
- Asymptotic analysis of reflection and transmission coefficients
- Future work

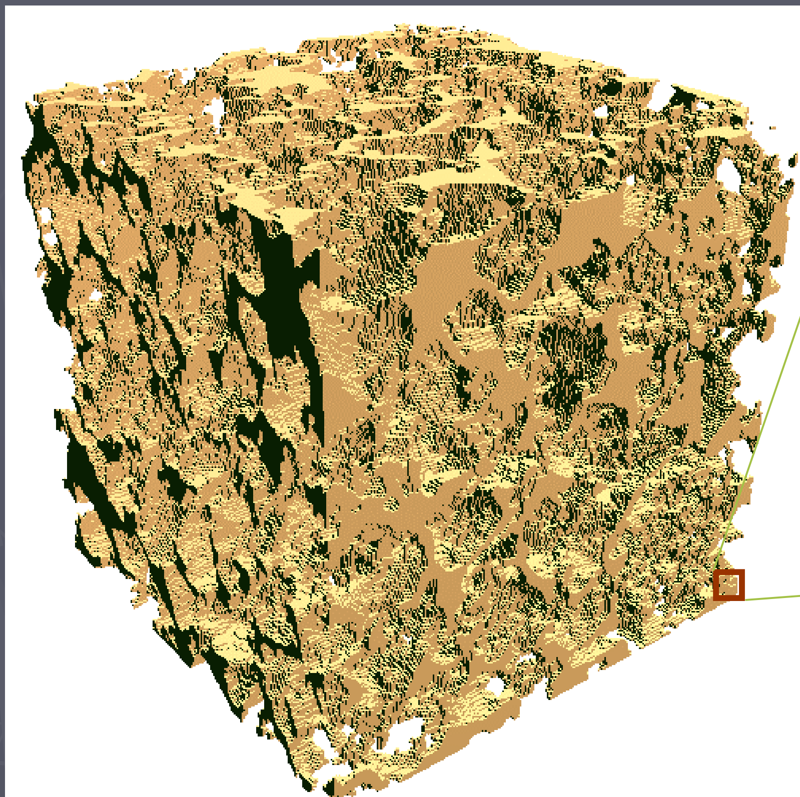
Fractured rock

Local heterogeneities: fractures and matrix

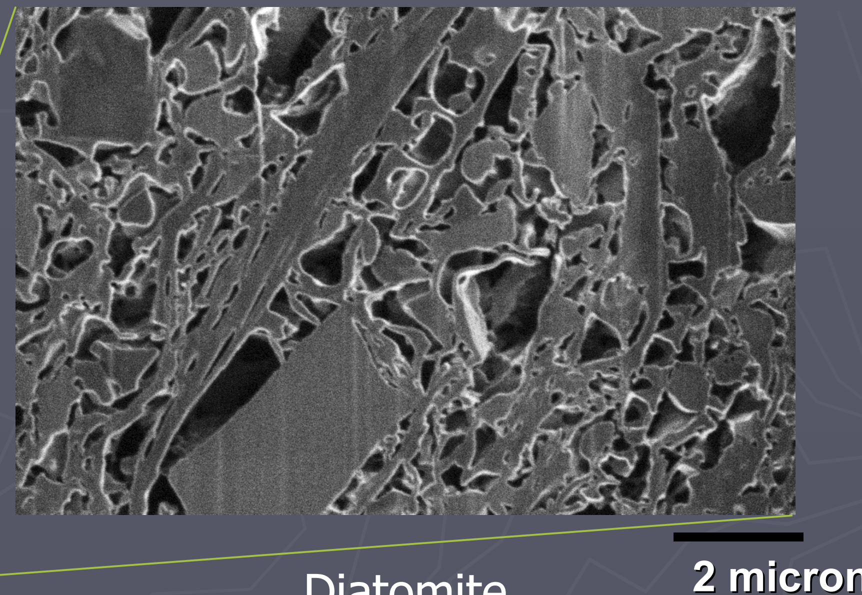


Source: Markus Tuller and Dani Or
Vadose Zone Journal 1:14–37 (2002)

Matrix



Berea sandstone

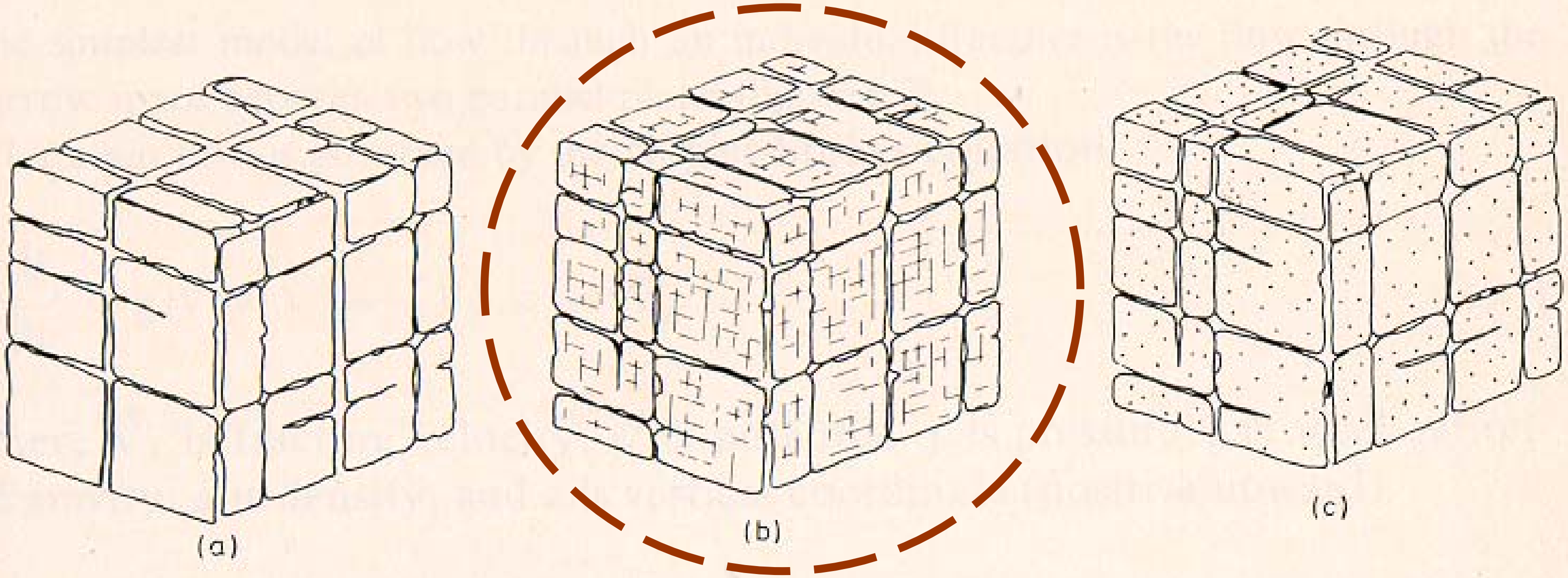


Diatomite

Matrix permeability is small due to the complexity of the pore space geometry

Images: Liviu Tomutsa LBNL (ALS and FIB)

Double-porosity model



6.1 – Sample of fractured reservoir Single porosity. (a) double porosity due to a system of microfissures and joints in the blocks (b), and double porosity due to granular porosity of the blocks.

T. D. van Golf-Racht, Fundamentals of fractured reservoir engineering, Elsevier, Amsterdam, 1982.

Double-porosity model: some history

- Barenblatt, G. I., Zheltov, Yu. P., and Kochina, I. N. (1960), basic concepts in the theory of seepage of homogeneous liquids in fissured rocks. J. Applied Mathematics and Mechanics (PMM), **24**, No. 5, pp. 1286-1303 (English Translation from Russian).
- J. E. Warren and Root, P. J.: "The Behavior of Naturally Fractured Reservoirs," SPEJ, (Sept. 1963) pp. 245-255
- S. R. Pride, J. G. Berryman. Linear dynamics of double-porosity dual-permeability materials. I - II. Phys. Rev. E **68**, (2003)

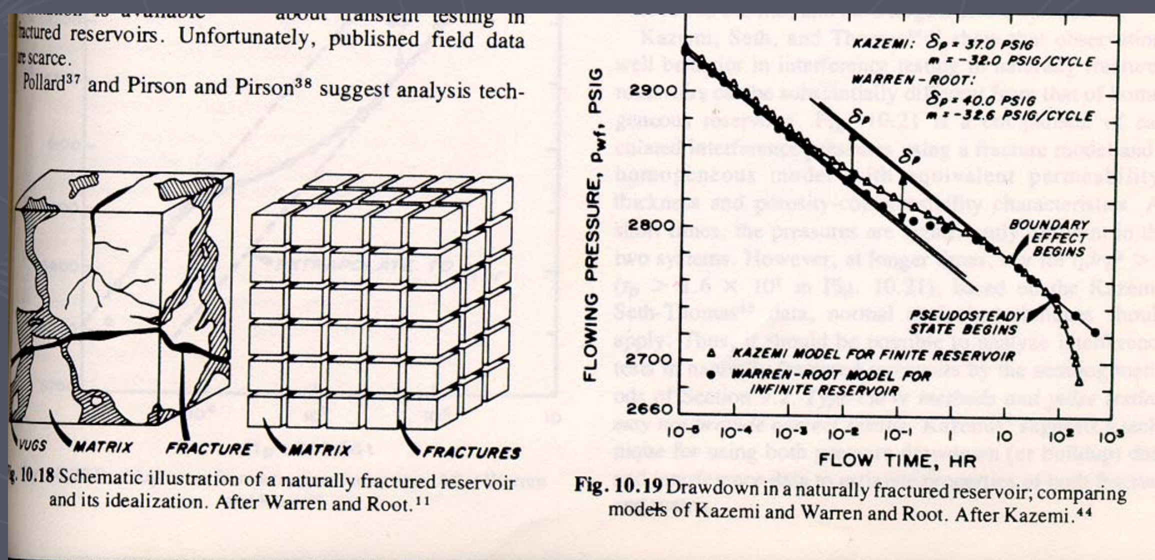


Fig. 10.18 Schematic illustration of a naturally fractured reservoir and its idealization. After Warren and Root.[11]

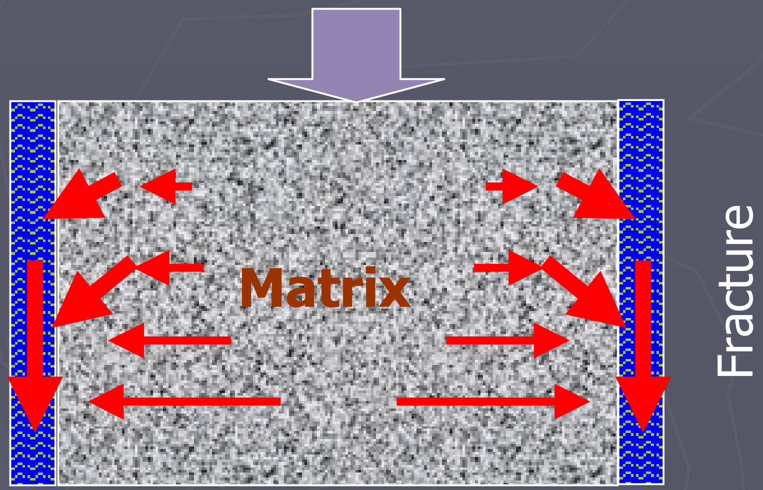
Fig. 10.19 Drawdown in a naturally fractured reservoir; comparing models of Kazemi and Warren and Root. After Kazemi.[44]

Dual-Medium Approach

- In every REV, two types of media are presented *simultaneously*:
  - *Matrix*: stores the fluid but only allows for fluid exchange with the surrounding fractures
  - *Connected system of fractures*: practically zero storage capacity, but fluid flow due to the simple geometry
  - Two fluid pressures are associated with each point in the medium
- The rate of fluid exchange is proportional to the *difference* between the fluid pressures in the two media

Double-porosity medium

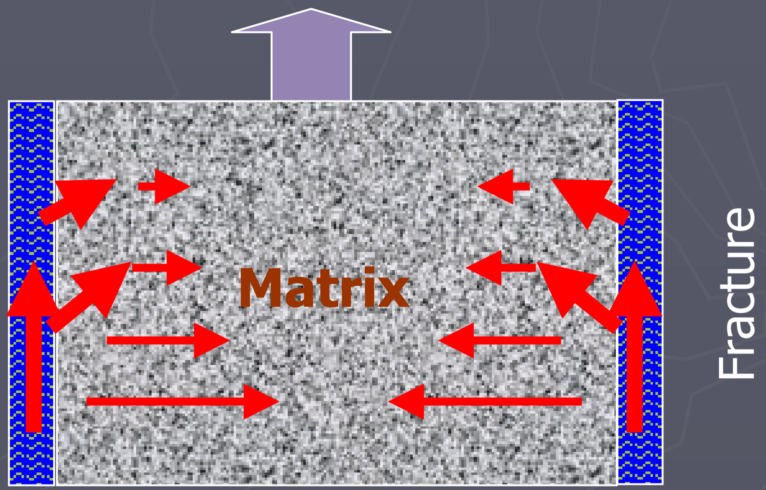
Compression



High matrix pressure

Low fracture pressure

Dilation



Low matrix pressure

High fracture pressure

Fluid flow and compressibility

$$W + \tau \frac{\partial W}{\partial t} = -\frac{\kappa}{\eta}\left(\frac{\partial p_f}{\partial x} + \rho_f \frac{\partial^2 u}{\partial t^2}\right)$$

Fractures: dynamic *Darcy's law*

$$q + \tau_{fm} \frac{\partial q}{\partial t} = -\frac{A}{\eta}\left(p_f - p_m\right)$$

Matrix-Fractures flow: A *-> shape factor*

$$\frac{d\rho}{\rho} = \beta_f dp \quad \left(= -\frac{dV}{V}\right)$$

Fluid compressibility: *slightly compressible* fluid

$$\frac{\partial u}{\partial x} = -\beta\sigma$$

Drained solid: linear elasticity

Mass Balance

$$q = \frac{\partial W}{\partial x}$$

Fractures: no fluid accumulation

$$\phi_m \beta_f \frac{\partial p_m}{\partial t} + \frac{\partial \phi_m}{\partial t} = -\phi_m \frac{\partial^2 u}{\partial t \partial x} - q$$

Matrix: fluid accumulation exclusively due to compression and deformation

$$\frac{\partial \phi_m}{\partial t} = \left(1 - \phi_m\right) \frac{\partial^2 u}{\partial t \partial x}$$

Solid: deformation results in porosity variation

Momentum balance

$$\frac{1}{v_b^2}\frac{\partial^2 u}{\partial t^2}+\frac{1}{v_f^2}\frac{\partial W}{\partial t}$$

Acceleration of fluid-solid system

$$=\frac{\partial^2 u}{\partial x^2}-\alpha_m\beta\frac{\partial p_m}{\partial x}-\alpha_f\beta\frac{\partial p_f}{\partial x}$$

Total stress

$$v_b^2=\frac{1}{\beta\rho_b}\qquad v_f^2=\frac{1}{\beta\rho_f}$$

Two modified speeds of sound

Harmonic wave solution: asymptotic form

$$u = U^{S} e^{i(\omega t - k_S x)} + U^{F} e^{i(\omega t - k_F x)}$$

Slow + fast wave

$$\varepsilon = i \frac{\rho_f \kappa \omega}{\eta}$$

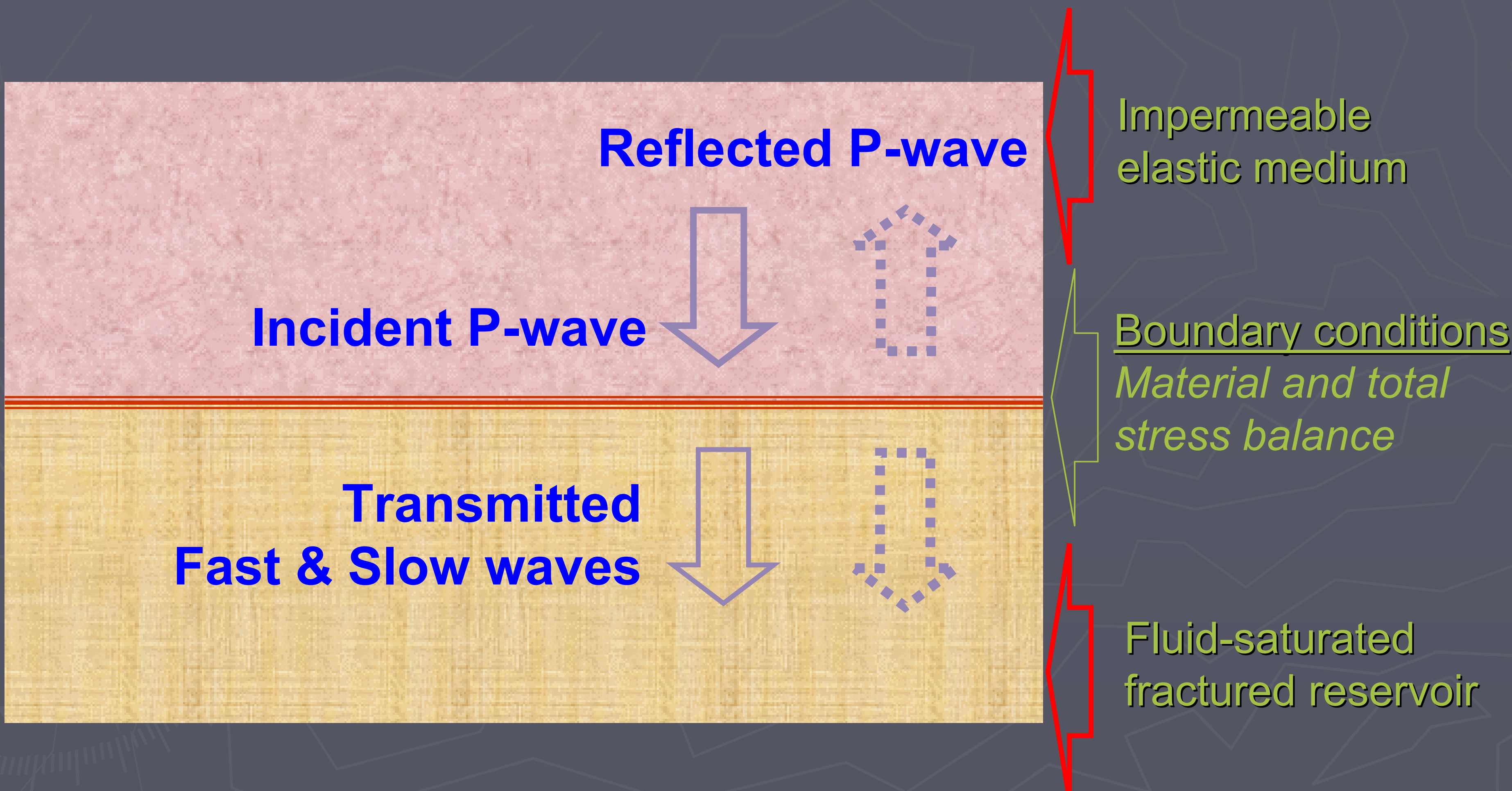
Small (dimensionless) parameter

$$a^{F} = \varsigma^{F} \frac{1 + O(\varepsilon)}{v_f} |\varepsilon| \omega$$

$$a^{S} = \varsigma^{S} \frac{1 + O(\varepsilon)}{v_f \sqrt{|\varepsilon|}} \omega$$

Example: attenuation factors

Reflection from impermeable top of the reservoir



Reflection from impermeable top of the reservoir

$$R = R_0 + R_1\sqrt{\varepsilon} + O(\varepsilon)$$

Reflection coefficient

$$T^F = T_0^F + T_1^F\sqrt{\varepsilon} + O(\varepsilon)$$

Transmission coefficient (fast)

$$T^S = O(\varepsilon)$$

Transmission coefficient (slow)

$$R_0 = \frac{Z_1 - Z_2^F}{Z_1 + Z_2^F} \qquad T_0^F = \frac{2Z_1}{Z_1 + Z_2^F}$$

Z_1 – impedance of the impermeable medium

$$R_1 = T_1^F = \varsigma T_1^F \frac{Z_2^S}{Z_1 + Z_2^F}$$

Z_2^F, Z_2^S – fast and slow impedances of the reservoir formation

Conclusions

- Low-frequency asymptotic analysis of elastic wave propagation and reflection in fractured fluid-saturated reservoir implies:
  - The fast wave attenuation factor is small of higher order relative to that of slow wave
  - The main frequency-dependent components of reflection and fast transmission coefficients are proportional to the square root of frequency
  - The frequency-independent component of the slow transmission coefficient is zero

Future plans

- Analyze reflection from a finite-thickness reservoir
- Include internal reservoir heterogeneities
- Extend analysis to not normal incident wave (frequency-dependent AVO)

Acknowledgments

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