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LOW-FREQUENCY ASYMPTOTIC ANALYSIS OF REFLECTION COEFFICIENT FROM A HYDROCARBON RESERVOIR

Silin, D. B., Goloshubin, G. M., Korneev, V. A., and Patzek, T. W.

University of California, Berkeley
Berkeley, CA, 94720, U.S.A.
University of Houston
Lawrence Berkeley National Laboratory
e-mail: silin@patzek.berkeley.edu

ABSTRACT

Asymptotic low-frequency analysis of the planar p-wave reflection coefficient from a hydrocarbon reservoir shows that the frequency-dependent component is proportional to the square root of the frequency multiplied by the kinematic mobility of the reservoir fluid. The latter is defined as the product of the fluid mobility and density. This asymptotic scaling rule holds both for conventional poroelastic media and reservoirs with dual permeability.
Frequency-dependent reservoir imaging has been successfully applied both in on-shore and off-shore environments. As the obtained asymptotic scaling links reservoir rock and fluid properties with seismic attributes, it has a great potential both for hydrocarbon exploration and reservoir characterization in produced oil fields.

INRODUCTION

It has been recognized that seismic wave propagation in fluid-saturated porous media is strongly affected by the fluid properties and rock permeability. There are numerous examples of successful oil and gas reservoir imaging based on frequency-dependent analysis of reflected signal, (Goloshubin *et al.*, 1998-2005, Korneev *et al.*, 2004ab). An example of frequency-dependent seismic imaging is presented in Figure 1. The yellow spots correspond to the most permeable parts of an off-shore reservoir. Conventional analysis (not presented here) has not produced any noticeable contrast.
The dependence of the attenuation and reflection coefficients on the frequency of the signal suggests that the presence of viscous pore fluid requires use of poroelastic theory for adequate interpretation of the results. The fundamental theory of elastic wave propagation in a fluid-saturated porous rock has been developed by Biot (1956ab). Wave propagation in rocks with two scales of permeability was analyzed by Pride and Berryman (2003ab). Evaluation of the reflection coefficient from gas-water contact has been presented by Dutta and Ode (1983).

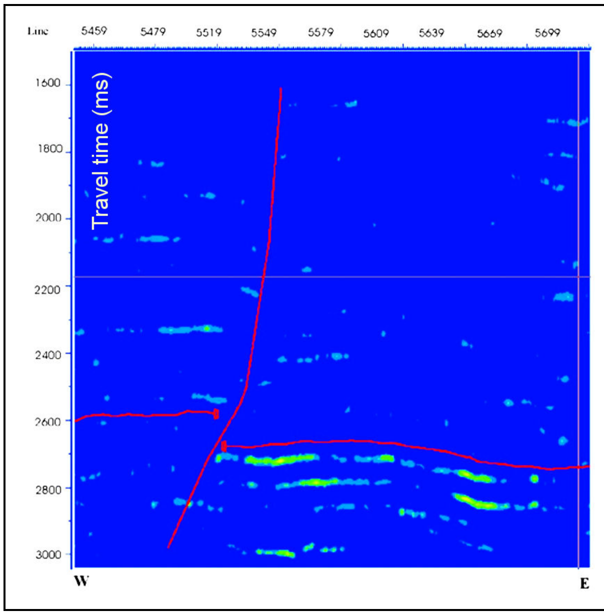

Figure 1. Imaging of the hydrocarbon-bearing zones in South Marsh Island, Gulf of Mexico, based on frequency-dependent analysis of 3D seismic data. Conventional AVO analysis did not detect the reservoir. Data are courtesy of Fairfield Industries.

In this presentation, we develop asymptotic analysis of the reflection coefficient in the low-frequency range of seismic spectrum. For this purpose, we obtain the governing equations in a dimensionless form and use $\varepsilon = i\frac{\rho_f \kappa \omega}{\eta}$ as the small dimensionless parameter in our asymptotic analysis. Here ρ_f is the density of reservoir fluid, κ is the reservoir rock permeability, η is the fluid viscosity, ω is the angular frequency of the signal and i is the square root of -1.
In the following Sections, we present asymptotic analysis of a harmonic-wave solution to the governing equations and obtain a simple expression of the planar p-wave reflection coefficient. Although this analysis and the conclusions hold both for conventional (Silin *et al.*, 2005) and dual (Goloshubin and Silin, 2005) porous media, here we focus on the former case due to the limited space.

Note that our dual-medium treatment is based on the model by Barenblatt *et al.* (1960) routinely used in reservoir engineering.

THE MODEL

Governing Equations

Denote by u the skeleton displacement, W the Darcy fluid velocity, and p the fluid pressure. Then, from the basic principles of filtration theory and linear elasticity, one obtains

$$\begin{aligned}\frac{\partial^2 u}{\partial t^2}+\gamma_\rho\frac{\partial W}{\partial t} &= v_b^2\frac{\partial^2 u}{\partial x^2}-v_f^2\frac{\partial P}{\partial x}\\ \rho_f\frac{\kappa}{\eta}\frac{\partial^2 u}{\partial t^2}+W+\tau\frac{\partial W}{\partial t} &= -D\frac{\partial P}{\partial x}\\ \frac{\partial^2 u}{\partial t\partial x}+\frac{\partial P}{\partial t} &= -\frac{\partial W}{\partial x}\end{aligned} \quad (1)$$

Here t is time and x is the coordinate aligned with wave propagation; γ_ρ is the ratio of the fluid density ρ_f and the saturated-medium bulk density ρ_b; $D=\dfrac{\kappa}{\phi\eta\beta_f}$ is the hydraulic diffusivity, where ϕ is the reservoir rock porosity; and β_f is the adiabatic fluid compressibility. The characteristic velocities v_b and v_f are defined as

$$v_f^2=\frac{1}{\beta\rho_b} \quad \text{and} \quad v_b^2=\frac{1}{\phi\beta_f\rho_b} \quad (2)$$

and $P=\phi\beta_f p$. It is assumed that the grain compressibility is negligible with respect to that of the drained skeleton, which is denoted by β. The first equation is the momentum balance based on elastic and effective stresses, the second one describes fluid flow in the reference system attached to the skeleton, and the last one is the fluid mass conservation. Note that the second equation involves relaxation time τ, which comes from a dynamic version of Darcy's law (Alishaev and Mirzadzhanzadeh, 1975). The relationship of equations (1) to classical Biot's model, as well as the relationship between the relaxation time and Biot's tortuosity parameter, is discussed in more detail in (Silin *et al*. 2005). Both velocities v_f and v_b are different from the speed of sound in the drained skeleton or bulk fluid. However, they are convenient parameters for the asymptotic analysis of attenuation and reflection.

Asymptotic Harmonic Wave Solution

We seek a solution to system of equations (1) in the form of harmonic plane wave

$$u=U_0e^{i(\omega t-kx)},\quad W=W_0e^{i(\omega t-kx)},\quad P=P_0e^{i(\omega t-kx)} \quad (3)$$

where k is the complex wave number yet to be determined. For asymptotic analysis, the following dimensionless variables are introduced

$$\varsigma=\frac{\omega^2}{k^2v_f^2},\quad \xi=-i\frac{P_0}{kU_0},\quad \chi=i\frac{W_0}{\omega U_0} \quad (4)$$

Then, equations (1) take on the following dimensionless form

$$\begin{aligned}-\varsigma+\xi+\gamma_\rho\varsigma\xi &= -\gamma_v\\ \varepsilon\left(-\varsigma+\frac{1}{\gamma_\rho}\xi+\theta\varsigma\chi\right)+\varsigma\chi &= 0\\ \xi+\chi &= 1\end{aligned} \quad (5)$$

Where $\gamma_v=\dfrac{\phi\beta_f}{\beta}$ and $\theta=\dfrac{\eta\tau}{\rho_f\kappa}$. In this study, we assume that $\theta=O(1)$. We seek an asymptotic solution as power series with respect to the small parameter ε:

$$\begin{aligned}\varsigma&=\varsigma_0+\varsigma_1\varepsilon+o(\varepsilon),\\ \xi&=\xi_0+\xi_1\varepsilon+o(\varepsilon),\\ \chi&=\chi_0+\chi_1\varepsilon+o(\varepsilon)\end{aligned} \quad (6)$$

Two solutions corresponding to the slow and fast waves are

$$\begin{aligned}\varsigma_0^S&=0 & \varsigma_1^S&=\gamma_v\delta\\ \xi_0^S&=-\gamma_v & \xi_1^S&=\gamma_v(\delta-1)\\ \chi_0^S&=1+\gamma_v & \chi_1^S&=\gamma_v(1-\delta)\end{aligned} \quad (7)$$

$$\begin{aligned}\varsigma_0^F&=1+\gamma_v & \varsigma_1^F&=\delta+\frac{1}{\delta}-2\\ \xi_0^F&=1 & \xi_1^F&=\delta-1\\ \chi_0^S&=0 & \chi_1^F&=1-\delta\end{aligned} \quad (8)$$

Where $\delta=\dfrac{1}{\gamma_\rho(1+\gamma_v)}$. Note that all coefficients in Equations (7)-(8) are real. Equations (4) and (7)-(8), in particular, imply that the slow wave propagates in the fluid, whereas the fast wave does not involve the fluid motion. In addition, the first asymptotic terms of the wave number and attenuation factor of the slow wave are equal to each other and asymptotically proportional to $\sqrt{\omega}$:

$$a^S \approx k^S = \frac{\omega}{v_b\sqrt{2\delta|\varepsilon|}} + \omega O\left(|\varepsilon|^{1/2}\right) \quad (9)$$

For the fast wave, the wave number and attenuation factor asymptotic expressions are

$$k^F = \frac{\omega}{\sqrt{v_f^2 + v_b^2}} + \omega O\left(|\varepsilon|\right) \text{ and } a^F = O\left(|\varepsilon|\right) \quad (10)$$

Thus, the slow wave attenuation factor, as a function of the frequency, is of higher order than that of the fast wave.

REFLECTION COEFFICIENT

Consider a plane interface between two media: one is an overburden formation and the other a fluid-saturated reservoir. The overburden formation is modeled as an elastic medium with the density ρ_1 and compressibility β_1. The speed of sound in this medium is defined as $v_1 = \sqrt{(\rho_1\beta_1)^{-1}}$. For the reservoir, we adopt the poroelastic model. An incident wave arriving at the interface between the two media is partially reflected and partially transmitted. Asymptotic analysis performed in the previous section can now be extended to investigate the dependence of the reflection coefficient on frequency. The transmitted wave has two components: the slow and the fast one. We will denote by u_1 and u_2 the skeleton displacement in the overburden and the reservoir rock, respectively. Mass and momentum conservation at the interface lead to the following three boundary conditions:

$$\begin{aligned} u_1 &= u_2 \\ -\frac{1}{\beta_1}\frac{\partial u_1}{\partial x}\bigg|_{x=0} &= -\frac{1}{\beta_2}\frac{\partial u_2}{\partial x}\bigg|_{x=0} + \phi p\big|_{x=0} \\ W\big|_{x=0} &= 0 \end{aligned} \quad (11)$$

Let U_0 be the amplitude of the incident wave. Then the total displacement in the overburden is $U_0 e^{i(\omega t - k_1 x)} + RU_0 e^{i(\omega t + k_1 x)}$, where R is the reflection coefficient. In the reservoir, the slow and fast wave have the amplitudes $T^S U_0$ and $T^F U_0$, respectively. Here T^S and T^F are the transmission coefficients for the slow and fast component of the transmitted wave. Applying Equations (4), (6)-(10), for the Darcy velocity and the fluid pressure amplitudes, one obtains

$$\begin{aligned} W_0^S &= -\left(1+\gamma_v\right)\omega T^S U^0 + \omega O\left(|\varepsilon|\right) \\ p_0^S &= \gamma_v \frac{1}{\phi\beta_f v_b}\sqrt{\frac{1}{\delta\varepsilon}} i\omega T^S U^0 + \omega O\left(|\varepsilon|^{1/2}\right) \end{aligned} \quad (12)$$

and

$$\begin{aligned} W_0^F &= \omega O\left(|\varepsilon|\right) \\ p_0^F &= -\frac{1}{\phi\beta_f\sqrt{v_f^2 + v_b^2}} i\omega T^S U^0 + \omega O\left(|\varepsilon|\right) \end{aligned} \quad (13)$$

Note that the assumption $\theta = O(1)$ puts the relaxation time (equivalently, the Biot's tortuosity factor) into the higher-order terms only. Substitution of Equations (12)-(13) into the boundary conditions (11) suggests that the asymptotic expansions of the reflection and transmission coefficients have the following forms

$$\begin{aligned} R &= R_0 + R_1\sqrt{\varepsilon} + o\left(\sqrt{|\varepsilon|}\right) \\ T^S &= T_0^S + T_1^S\varepsilon + o\left(|\varepsilon|\right) \\ T^F &= T_0^F + T_1^F\sqrt{\varepsilon} + o\left(\sqrt{|\varepsilon|}\right) \end{aligned} \quad (14)$$

Further analysis yields $T_0^S = 0$. Thus, the slow-wave transmission coefficient only includes higher-order terms. For the frequency-independent component of the reflection coefficient R_0, one obtains

$$R_0 = \frac{Z_2 - Z_1}{Z_2 + Z_1} \qquad T_0^F = \frac{2Z_2}{Z_2 + Z_1} \quad (15)$$

where $Z_1 = \sqrt{\frac{\rho_1}{\beta_1}}$ is the acoustic impedance of the overburden formation and

$$Z_2 = \left(\frac{1}{\beta_2} + \frac{1}{\beta_f}\right)\frac{1}{\sqrt{v_f^2 + v_b^2}} \quad (16)$$

is the "mixed" acoustic impedance of the fluid-saturated reservoir. For the second term, one obtains

$$R_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1}\frac{1-\delta}{1+\gamma_v} Z_3 T_0^F \quad (17)$$

where

$$Z_3 = \frac{1-\phi}{v_b\beta_2\sqrt{\delta}} \quad (18)$$

For a dual-medium reservoir model, the asymptotic analysis of the reflection coefficient leads to a formula qualitatively similar to the one in Equation (14), but with different expressions for the coefficients R_0 and R_1 (Goloshubin and Silin, 2005). The nature of this difference is the decoupling of the permeability and porosity in a dual medium.

CONCLUSIONS

Low-frequency, frequency-dependent analysis of seismic reflection leads to high-quality reservoir images. The fluid-saturated regions are accurately delineated even when standard analysis produces no results.

Asymptotic analysis of the reflection coefficient for plane p-wave shows that the frequency-dependent component is proportional to the square root of the product of signal frequency, and the reservoir fluid density and mobility. The reflection coefficient is a complex number that indicates additional phase shift.

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